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Spin-spin interaction in Matrix theory

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Abstract: We calculate the spin dependent static force between two D0-branes in Matrix theory. Supersymmetry relates velocity dependent potentials to spin dependent potentials. The well known v^4/r^7 term is related to a θ^8/r^{11} term, where θ is the relative spin of the D0-branes. We calculate this term, confirming that it is the lowest order contribution to the static potential, and find its structure consistient with supergravity.

Keywords: D-branes, M(atrix) Theories, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Matrix theory is conjectured to be M-theory in the large N limit [1]. Therefore, it should contain eleven dimensional supergravity at low energies. Even prior to this conjecture, it was argued that the leading order scattering amplitude of two gravitons can be calculated using the ten dimensional gauge theory of the string zero modes dimensionally reduced to the world lines of D0-particles, i.e. Matrix theory [2]. The spin independent interaction of D0-branes,proportional to v^4/r^7, has been calculated and was shown to be in agreement with eleven dimensional supergravity up to the two loop level [3, 4, 5].

There should be other terms related to the bosonic v^4/r^7 term by supersymmetry [6, 7]. Indeed, the power counting of the Matrix theory allows one to trade θ^2/r for each power of v, where θ is the relative spin degree of freedom. The first of these terms, $v^3\theta^2/r^8$, is a spin-orbit interaction and was calculated in [8], where agreement with supergravity was found. It has been argued, by comparing massless open string loops to massless closed string exchange, that all of these terms will agree [6]. In this paper we calculate the static force between D0-branes, expected to go like θ^8/r^{11}.

Recently, discrepancies between naïve matrix predictions and supergravity were found for three graviton scattering [9], for Matrix theory on ALE spaces [10] and for $\mathcal{R}^4$ terms in supergravity that arise from one loop divergences [11]. These discrepancies are argued to be due to the fact that these are finite N calculations and that these should be compared to the compactification of eleven dimensional supergravity on a light-like circle [12, 13, 14]. We have little to add to this debate. The exact numerical agreement that was miraculously found for the two graviton amplitude is expected to persist in our calculation since it is simply a supersymmetric extension of that earlier calculation. It is our hope that the techniques presented in this paper will eventually prove useful for calculations involving more than two particles. These results could help clarify the relation between eleven dimensional supergravity and Matrix theory.

Calculating the effective potential is equivalent to calculating the effective action for a static configuration. There are two techniques for calculating the effective action. One is to treat the classical field (defined to be the expectation value of the quantum field) as a background field [15, 4]. Quantum corrections come from one-particle-irreducible vacuum diagrams. The other is to treat the classical field as a perturbation, expanding in powers of the field [16]. In this method the quantum corrections to the nth coefficient in the expansion are one-particle-irreducible diagrams with n external lines. In this paper we use both methods. The classical bosonic fields, representing the positions of the particles, are treated as a background while the fermionic fields, representing the particles' spin, are treated as perturbations. This is fairly natural since the fermionic degrees of freedom are anticommuting and therefore their power series expansion terminates after a finite number of terms (at least for finite N).

Following this philosophy we find the Feynman rules for the component fields of $N = 2$ Matrix theory. We show that all terms with $4k+2$ background fermions vanish. This is related by supersymmetry to the vanishing of odd powers of velocity in the spin independent case. Then, we work out the θ^4/r^5 potential. This vanishes in agreement to the vanishing of the corresponding bosonic term v^2/r^3. Finally, we find the first nonvanishing static contribution, which is proportional to θ^8/r^{11}.

2. The bosonic background

We start from the Matrix theory action

$$\mathcal{S}_{\mathrm{Matrix}} = \frac{1}{g_{\mathrm{s}} l_{\mathrm{s}}} \int dt \,\mathrm{Tr} \left(-(D_t \mathbf{X}^i)^2 + \frac{1}{2 l_{\mathrm{s}}^4} \left[\mathbf{X}^i, \mathbf{X}^j\right]^2 - i \mathbf{\Theta} D_t \mathbf{\Theta} + \frac{i}{l_{\mathrm{s}}^2} \mathbf{\Theta} \gamma^i \left[\mathbf{X}^i, \mathbf{\Theta}\right] \right), \quad (2.1)$$

where $D_t \mathbf{X}^i = \partial_t \mathbf{X}^i + [\mathbf{A}, \mathbf{X}^i]$.

Following the strategy outlined in the introduction, we split the bosonic fields into a classical part (obeying the classical equations of motion) and quantum fluctuations. The gauge invariance of the background field can be used to set $\mathbf{A}_{\mathrm{cl}} = 0$. The gauge fixing condition for $\mathbf{A}_{\mathrm{q}}$ comes from requiring the background covariant divergence of the gauge fields to vanish.

$$l_{\mathrm{s}}^2 \partial_t \mathbf{A}_{\mathrm{q}} - \frac{1}{l_{\mathrm{s}}^2} \left[\mathbf{X}^i_{\mathrm{cl}}, \mathbf{X}^i_{\mathrm{q}}\right] = 0\,.$$

This leads to a gauge fixing term.

$$\mathcal{L}_{\rm gf} = \frac{1}{g_{\rm s} l_{\rm s}} \,{\rm Tr} \left(l_{\rm s}^2 \partial_t \mathbf{A}_{\rm q} - \frac{1}{l_{\rm s}^2} \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^i_{\rm q} \right] \right)^2 . \tag{2.2}$$

The Lagrangian for the Faddeev-Popov ghosts can be found in [4]. However, it is not needed for the problem under consideration since the ghosts do not couple to the fermions at 1-loop order.

We separate the Lagrangian into classical and quantum parts.

$$\mathcal{L} = \frac{1}{g_{\rm s}} (\mathcal{L}_{\rm cl} + \mathcal{L}_X + \mathcal{L}_A + \mathcal{L}_\Theta + \mathcal{L}_{\rm ghost}) . \tag{2.3}$$

We require the background fields satisfy the classical field equations, so all terms containing a single quantum field vanish. We also use the identity

$${\rm Tr} \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^j_{\rm q} \right] \left[\mathbf{X}^i_{\rm q}, \mathbf{X}^j_{\rm cl} \right] + {\rm Tr} \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^i_{\rm q} \right]^2 = {\rm Tr} \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^j_{\rm cl} \right] \left[\mathbf{X}^i_{\rm q}, \mathbf{X}^j_{\rm q} \right] ,$$

which can be derived from ${\rm Tr}\, A\, [B, C] = {\rm Tr}\, [A, B]\, C$ and the Jacobi identity.

$$\begin{aligned}
\mathcal{L}_X &= {\rm Tr} \Bigg(-(\partial_t \mathbf{X}^i_{\rm q})^2 + \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^j_{\rm q} \right]^2 + 2 \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^j_{\rm cl} \right] \left[\mathbf{X}^i_{\rm q}, \mathbf{X}^j_{\rm q} \right] \\
&\qquad\qquad + 2 \left[\mathbf{X}^i_{\rm cl}, \mathbf{X}^j_{\rm q} \right] \left[\mathbf{X}^i_{\rm q}, \mathbf{X}^j_{\rm q} \right] + \frac{1}{2} \left[\mathbf{X}^i_{\rm q}, \mathbf{X}^j_{\rm q} \right]^2 \Bigg) \\
\mathcal{L}_A &= {\rm Tr} \Bigg((\partial_t \mathbf{A})^2 - 4 i \partial_t \mathbf{X}^i_{\rm cl} \left[\mathbf{A}, \mathbf{X}^i_{\rm q} \right] - \left[\mathbf{A}, \mathbf{X}^i_{\rm cl} \right]^2 \\
&\qquad\qquad - 2 i \partial_t \mathbf{X}^i_{\rm q} \left[\mathbf{A}, \mathbf{X}^i_{\rm q} \right] - 2 \left[\mathbf{A}, \mathbf{X}^i_{\rm cl} \right] \left[\mathbf{A}, \mathbf{X}^i_{\rm q} \right] - \left[\mathbf{A}, \mathbf{X}^i_{\rm q} \right]^2 \Bigg) \\
\mathcal{L}_\Theta &= {\rm Tr} \left(-i \mathbf{\Theta} \partial_t \mathbf{\Theta} - i \mathbf{\Theta} \left[\mathbf{A}, \mathbf{\Theta} \right] + i \mathbf{\Theta} \gamma^i \left[\mathbf{X}^i_{\rm cl}, \mathbf{\Theta} \right] + i \mathbf{\Theta} \gamma^i \left[\mathbf{X}^i_{\rm q}, \mathbf{\Theta} \right] \right) .
\end{aligned} \tag{2.4}$$

2.1. Component fields for $N = 2$

We are interested in the static force between two D0-branes in the center of mass system. Therefore we consider only a static background of fields in the Cartan subalgebra.

$$\begin{aligned}
\mathbf{X}^i_{\rm cl} &= \frac{i}{2} \begin{bmatrix} r^i & 0 \\ 0 & -r^i \end{bmatrix} & \qquad \mathbf{X}^i_{\rm q} &= \frac{i}{2} \begin{bmatrix} X^i & \sqrt{2}\, \overline{Y}^i \\ \sqrt{2}\, Y^i & -X^i \end{bmatrix} \\
\mathbf{A} &= \frac{i}{2} \begin{bmatrix} A & \sqrt{2}\, \overline{B} \\ \sqrt{2}\, B & -A \end{bmatrix} & \qquad \mathbf{\Theta} &= \frac{i}{2} \begin{bmatrix} \theta & \sqrt{2}\, \overline{\psi} \\ \sqrt{2}\, \psi & -\theta \end{bmatrix} .
\end{aligned}$$

The bar represents complex conjugation. All of the fields on the diagonals are real, while the off diagonal fields are complex.

The Lagrangian can be written in terms of these real and complex fields. Only interactions involving fermions will be necessary for this calculation, so purely bosonic

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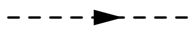
interactions are left out of the Lagrangians.

$$\begin{aligned}\mathcal{L}_X &= -\tfrac{1}{2}X^i\partial_t^2X^i - \overline{Y}^i(\partial_t^2+r^2)Y^i + (\text{interactions})\\ \mathcal{L}_A &= \tfrac{1}{2}A\partial_t^2A + \overline{B}(\partial_t^2+r^2)B + (\text{interactions})\\ \mathcal{L}_\Theta &= \tfrac{1}{2}\theta i\partial_t\theta + \overline{\psi}(i\partial_t - \not{r})\psi + Y^i\overline{\psi}\gamma^i\theta + \overline{Y}^i\theta\gamma^i\psi - B\overline{\psi}\theta - \overline{B}\theta\psi - X^i\overline{\psi}\gamma^i\psi + A\overline{\psi}\psi\end{aligned} \tag{2.5}$$

We will integrate out the off diagonal fields to get an effective action for the diagonal fields. The background has given a mass to the off diagonal fields, which will prevent infrared divergences.

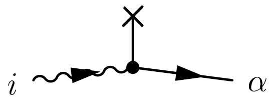
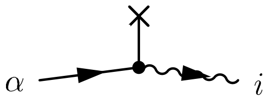
2.2. Feynman rules

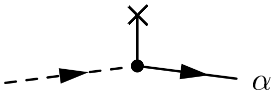
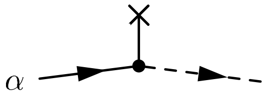
The Feynman rules can easily be read off of the Lagrangian. The off diagonal fields will be integrated out, so only their propagators are needed.

$\left\langle \overline{Y}^i Y^j \right\rangle = j$  $i = \dfrac{i\delta^{ij}}{\omega^2 - r^2 + i\epsilon}$ $\qquad \left\langle \overline{B}B \right\rangle =$  $= \dfrac{-i}{\omega^2 - r^2 + i\epsilon}$

$\left\langle \overline{\psi}\psi \right\rangle = \beta$  $\alpha = i\dfrac{(\omega + \not{r})_{\alpha\beta}}{\omega^2 - r^2 + i\epsilon}\,.$

The spin of the D0-branes enters the calculation through the interaction of the above massive fields with the θ field. The vertices (with the external θ attached, as explained below) are

 $= i\gamma^i_{\alpha\beta}\theta_\beta$  $= i\theta_\beta\gamma^i_{\beta\alpha}$

 $= -i\theta_\alpha$  $= -i\theta_\alpha\,.$

3. The expansion in θ

As described in the introduction, we expand the effective action in powers of θ.

$$\Gamma(r,\theta) = \sum_n \int dt_1\cdots dt_n\,\Gamma^{(n)}_{\alpha_1,\ldots,\alpha_n}(r;t_1,\ldots,t_n)\,\theta_{\alpha_1}(t_1)\cdots\theta_{\alpha_n}(t_n)\,. \tag{3.1}$$

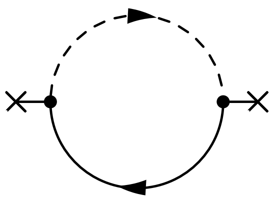
It turns out that $i\Gamma^{(n)}_{\alpha_1,\ldots,\alpha_n}(r;t_1,\ldots,t_n)$ is the sum of all one-particle-irreducible diagrams with n external fermion lines. In frequency space, the effective potential is just minus the effective action evaluated at zero momentum:

$$\begin{aligned} V_{\text{eff}}(r,\theta) &= -\sum_n \widetilde{\Gamma}^{(n)}_{\alpha_1,\ldots,\alpha_n}(r;0,\ldots,0)\,\theta_{\alpha_1}\cdots\theta_{\alpha_n} \\ &= i\sum_n (\text{diagrams})\,, \end{aligned} \tag{3.2}$$

where the diagrams are the one-particle-irreducible diagrams with n, zero-frequency external lines with θ's attached.

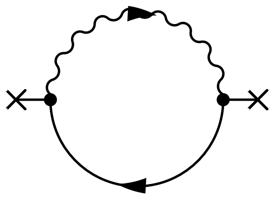
3.1. Terms proportional to θ^{4k+2}

The θ^2 term is easy, since $\theta\theta = \theta\gamma^i\theta = 0$. First, the diagram with the gauge field.



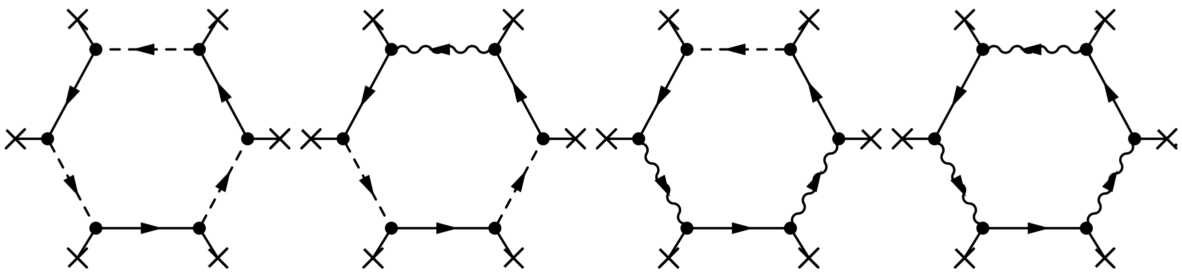
$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{-\theta(\omega + \not{r})\theta}{(\omega^2 - r^2 + i\epsilon)^2} = 0\,. \tag{3.3}$$

In fact any diagram that has a fermion between two gauge fields vanishes. Next, the one with the Y fields.



$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta\gamma^i(\omega + \not{r})\gamma^i\theta}{(\omega^2 - r^2 + i\epsilon)^2} = 0\,. \tag{3.4}$$

Both of the above diagrams are traversed by only one fermion. We will call any series of fermion traversals connected by vectors (Y fields) a *chain*. A chain is ended by scalars (B fields), or it connects back on its self (when the loop contains no scalars). As an example, there are four θ^6 diagrams:



The first contains three chains with only a single fermion traversing each (one link chains). The second contains a one link and a two link chain. The last two contain three link chains.

Here we prove that any diagram containing a chain with an odd number of links is zero. First, consider a closed chain with n links. Before doing the integral over ω, it will contain a factor

$$\theta\gamma^{i_1}(\omega+\not{v})\gamma^{i_2}\theta\,\theta\gamma^{i_2}(\omega+\not{v})\gamma^{i_3}\theta\,\cdots\theta\gamma^{i_{n-1}}(\omega+\not{v})\gamma^{i_n}\theta\,\theta\gamma^{i_n}(\omega+\not{v})\gamma^{i_1}\theta\,.$$

Each fermion bilinear is antisymmetric in its vector indices. Since there are an odd number of these factors, swapping the indices on all of them produces an overall minus sign.

$$-\theta\gamma^{i_2}(\omega+\not{v})\gamma^{i_1}\theta\,\theta\gamma^{i_3}(\omega+\not{v})\gamma^{i_2}\theta\,\cdots\theta\gamma^{i_n}(\omega+\not{v})\gamma^{i_{n-1}}\theta\,\theta\gamma^{i_1}(\omega+\not{v})\gamma^{i_n}\theta\,.$$

The factors can then be reordered,

$$-\theta\gamma^{i_1}(\omega+\not{v})\gamma^{i_n}\theta\,\theta\gamma^{i_n}(\omega+\not{v})\gamma^{i_{n-1}}\theta\,\cdots\theta\gamma^{i_3}(\omega+\not{v})\gamma^{i_2}\theta\,\theta\gamma^{i_2}(\omega+\not{v})\gamma^{i_1}\theta\,,$$

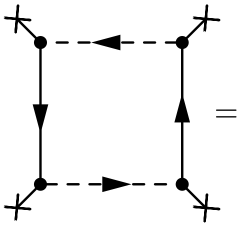
to reproduce the original expression, but with a minus sign (and the indices renamed). This implies that the term is zero. Chains ended by scalars contribute a factor

$$\theta\not{v}\gamma^{i_1}\theta\,\theta\gamma^{i_1}(\omega+\not{v})\gamma^{i_2}\theta\,\cdots\theta\gamma^{i_{n-2}}(\omega+\not{v})\gamma^{i_{n-1}}\theta\,\theta\gamma^{i_{n-1}}\not{v}\theta\,,$$

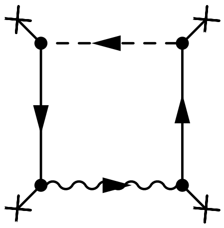
which vanishes for similar reasons. Therefore, any diagram containing a chain with an odd number of links is zero. All diagrams that are order 2 (mod 4) in θ have an odd number of fermions traversing them, so they must contain a chain with an odd number of links. For a diagram to give a nonvanishing contribution, the number of external θ lines must be a multiple of four. The diagrams with θ^{4k+2} are related by supersymmetry to bosonic diagrams with v^{2k+1}, which are required to vanish by time-reversal symmetry.

3.2. The vanishing θ^4 term

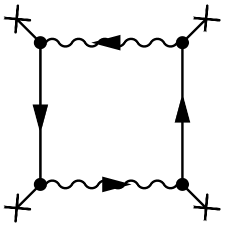
The θ^4 term is also zero, but demonstrating this is more difficult. There is one diagram that vanishes because it contains odd chains.

 $= 0\,.$

The cancellation of the remaining two diagrams requires two identities, (A.3) and (A.4), which are derived in the appendix.



$$
\begin{aligned}
&= -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta(\omega + \not{v})\gamma^i\theta\, \theta\gamma^i(\omega + \not{v})\theta}{(\omega^2 - r^2 + i\epsilon)^4} \\
&= -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta\not{v}\gamma^i\theta\, \theta\gamma^i\not{v}\theta}{(\omega^2 - r^2 + i\epsilon)^4}
\end{aligned}
\tag{3.5}
$$

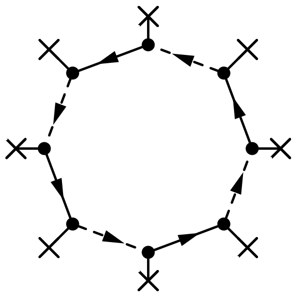
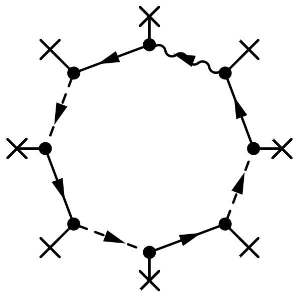
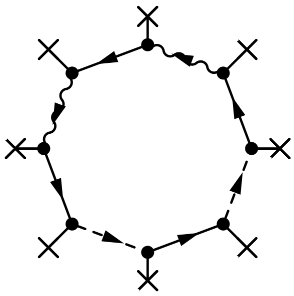


$$
\begin{aligned}
&= \frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta\gamma^i(\omega + \not{v})\gamma^j\theta\, \theta\gamma^j(\omega + \not{v})\gamma^i\theta}{(\omega^2 - r^2 + i\epsilon)^4} \\
&= \frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega^2\theta\gamma^i\gamma^j\theta\, \theta\gamma^j\gamma^i\theta + \theta\gamma^i\not{v}\gamma^j\theta\, \theta\gamma^j\not{v}\gamma^i\theta}{(\omega^2 - r^2 + i\epsilon)^4} \\
&= \frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta\gamma^i\not{v}\gamma^j\theta\, \theta\gamma^j\not{v}\gamma^i\theta}{(\omega^2 - r^2 + i\epsilon)^4} \\
&= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta\not{v}\gamma^i\theta\, \theta\gamma^i\not{v}\theta}{(\omega^2 - r^2 + i\epsilon)^4}\,.
\end{aligned}
\tag{3.6}
$$

Again, the vanishing of this term is related via supersymmetry to the vanishing of a purely bosonic term, namely v^2/r^5.

3.3. The θ^8 term

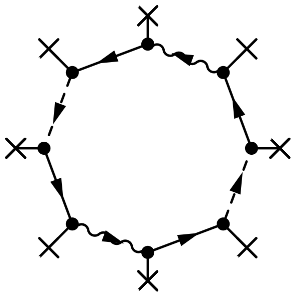
The θ^8 term should give the first non-zero contribution since it corresponds to the well known v^4/r^7 term. There are three diagrams that vanish due to odd chains:

 $=$  $=$  $= 0\,.$

To calculate the remaining diagrams we will need the following integrals.

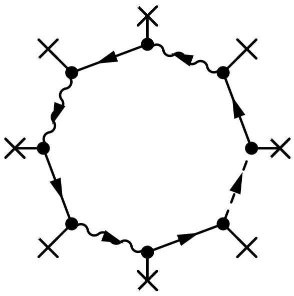
$$
\begin{aligned}
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(\omega^2 - r^2 + i\epsilon)^8} &\overset{\epsilon \to 0}{=} \frac{429i}{4096 r^{15}} \\
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega^2}{(\omega^2 - r^2 + i\epsilon)^8} &\overset{\epsilon \to 0}{=} -\frac{33i}{4096 r^{13}} \\
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega^4}{(\omega^2 - r^2 + i\epsilon)^8} &\overset{\epsilon \to 0}{=} \frac{9i}{4096 r^{11}} .
\end{aligned}
$$

The first diagram contains two B and two Y^i fields in the loop.



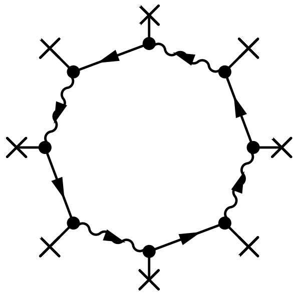
$$
\begin{aligned}
&= \frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{(\theta \not{r} \gamma^i \theta \, \theta \gamma^i \not{r} \theta)^2}{(\omega^2 - r^2 + i\epsilon)^8} \\
&= \frac{429i}{8192 r^{15}} \left(\theta \not{r} \gamma^i \theta \, \theta \gamma^i \not{r} \theta\right)^2 .
\end{aligned} \tag{3.7}
$$

To simplify the two final diagrams we employ identities (A.7), (A.8), (A.9) and (A.11) from the appendix. With these we obtain



$$
\begin{aligned}
&= -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\theta \not{r} \gamma^i \theta \, \theta \gamma^i (\omega + \not{r}) \gamma^j \theta \, \theta \gamma^j (\omega + \not{r}) \gamma^k \theta \, \theta \gamma^k \not{r} \theta}{(\omega^2 - r^2 + i\epsilon)^8} \\
&= \frac{33i}{4096 r^{13}} \theta \not{r} \gamma^i \theta \, \theta \gamma^i \gamma^j \theta \, \theta \gamma^j \gamma^k \theta \, \theta \gamma^k \not{r} \theta \\
&\quad - \frac{429i}{4096 r^{15}} \theta \not{r} \gamma^i \theta \, \theta \gamma^i \not{r} \gamma^j \theta \, \theta \gamma^j \not{r} \gamma^k \theta \, \theta \gamma^k \not{r} \theta \\
&= -\frac{99i}{1024 r^{13}} \theta \not{r} \gamma^i \theta \, \theta \gamma^i \gamma^j \theta \, \theta \gamma^j \gamma^k \theta \, \theta \gamma^k \not{r} \theta \\
&\quad + \frac{429i}{1024 r^{15}} \left(\theta \not{r} \gamma^i \theta \, \theta \gamma^i \not{r} \theta\right)^2
\end{aligned} \tag{3.8}
$$

and, finally,

 $=$

$$
\begin{aligned}
&= \frac{1}{4}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\theta\gamma^i(\omega+\not{v})\gamma^j\theta\,\theta\gamma^j(\omega+\not{v})\gamma^k\theta\,\theta\gamma^k(\omega+\not{v})\gamma^l\theta\,\theta\gamma^l(\omega+\not{v})\gamma^i\theta}{(\omega^2-r^2+i\epsilon)^8} \\
&= \frac{9i}{16384r^{11}}\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\gamma^l\theta\,\theta\gamma^l\gamma^i\theta \\
&\quad - \frac{33i}{4096r^{13}}\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\not{v}\gamma^k\theta\,\theta\gamma^k\not{v}\gamma^l\theta\,\theta\gamma^l\gamma^i\theta \\
&\qquad - \frac{33i}{8192r^{13}}\theta\gamma^i\not{v}\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{v}\gamma^l\theta\,\theta\gamma^l\gamma^i\theta \\
&\qquad\quad + \frac{429i}{16384r^{15}}\theta\gamma^i\not{v}\gamma^j\theta\,\theta\gamma^j\not{v}\gamma^k\theta\,\theta\gamma^k\not{v}\gamma^l\theta\,\theta\gamma^l\not{v}\gamma^i\theta \\
&= \frac{15i}{1024r^{11}}\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\gamma^l\theta\,\theta\gamma^l\gamma^i\theta \\
&\quad - \frac{231i}{1024r^{13}}\theta\not{v}\gamma^i\theta\,\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{v}\theta \\
&\qquad + \frac{4719i}{8192r^{15}}\left(\theta\not{v}\gamma^i\theta\,\theta\gamma^i\not{v}\theta\right)^2 .
\end{aligned}
\tag{3.9}
$$

4. Conclusion

Summing the above diagrams as in (3.2) we find the effective potential at order θ^8/r^{11}:

$$
\begin{aligned}
V_{\text{eff}}(r,\theta) = -\frac{15}{(2r)^{11}}\Bigg(& 2\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\gamma^l\theta\,\theta\gamma^l\gamma^i\theta \\
& - \frac{44}{r^2}\theta\not{v}\gamma^i\theta\,\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{v}\theta \\
& + \frac{143}{r^4}\left(\theta\not{v}\gamma^i\theta\,\theta\gamma^i\not{v}\theta\right)^2\Bigg) \\
= -\frac{5}{43{,}008}\left(\theta\not{\partial}\gamma^i\theta\,\theta\gamma^i\not{\partial}\theta\right)^2\frac{1}{r^7} .
\end{aligned}
\tag{4.1}
$$

There may be terms higher order in θ. Since θ only has sixteen components, the only two terms that could remain would be proportional to θ^{12}/r^{17} and θ^{16}/r^{23}. These are not related to the v^4/r^7 term but to higher order v^6/r^{11} and v^8/r^{15} terms respectively.

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It is remarkable that the contributions from the diagrams conspire to give exactly the coefficient that one gets from acting on $1/r^7$ with four gradients.

In order to make contact with supergravity we take the Fourier transform.

$$V_{\text{eff}}(q,\theta) = -\frac{\pi^4}{252}\frac{(q^i J^{ij} J^{jk} q^k)^2}{q^2}\,, \tag{4.2}$$

where we have put in the the relative angular momentum of the D0-branes, $J^{ij} = \frac{i}{2}\theta\gamma^i\gamma^j\theta$ [8]. The q^2 in the denominator is characteristic for an exchanged graviton, and the structure is the same as the (eight dimensional) static supergravity result in [7].

We would like to thank J. Harvey for many useful discussions and P. Pouliot for a discussion of supersymmetry's role in relating the various potentials. After the completion of this work, a similar question was addressed using string scattering theory [17].

A. SO(9) spinor identities

Using the Clifford algebra relation $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$ one immediately obtains

$$\{\not{r}, \gamma^i\} = 2r^i \qquad\qquad \not{r}\not{r} = r^2$$
$$\gamma^i\gamma^{j_1 j_2\cdots j_n}\gamma^i = (-1)^n(9-2n)\gamma^{j_1 j_2\cdots j_n}\,,$$

where $\gamma^{i_1 i_2 \ldots i_n} \equiv \gamma^{[i_1}\gamma^{i_2}\cdots\gamma^{i_n]}$.

We use a representation of the $SO(9)$ Clifford algebra with real, symmetric Dirac matrices [18]. Therefore, antisymmetrized products of two and three γ-matrices are antisymmetric in the spinor indices whereas products of 0,1,4, and 5 are symmetric. From this it follows

$$\begin{aligned}
&\theta\theta = 0 && \theta\gamma^i\theta = 0\\
&\theta\gamma^i\gamma^j\theta = \theta\gamma^{ij}\theta && \theta\gamma^i\gamma^j\gamma^k\theta = \theta\gamma^{ijk}\theta\\
&\theta\gamma^i\gamma^j\gamma^k\gamma^l\theta = \delta^{ij}\theta\gamma^{kl}\theta - \delta^{ik}\theta\gamma^{jl}\theta + \delta^{il}\theta\gamma^{jk}\theta + \delta^{jk}\theta\gamma^{il}\theta - \delta^{jl}\theta\gamma^{ik}\theta + \delta^{kl}\theta\gamma^{ij}\theta\,.
\end{aligned} \tag{A.1}$$

We also make frequent use of the following Fierz identity, which can be derived from the fact that γ^{ij} and γ^{ijk} form a complete basis for 16×16 matrices antisymmetric in α and β:

$$\theta_\alpha\theta_\beta = \frac{1}{32}\theta\gamma^i\gamma^j\theta\,(\gamma^i\gamma^j)_{\alpha\beta} + \frac{1}{96}\theta\gamma^i\gamma^j\gamma^k\theta\,(\gamma^i\gamma^j\gamma^k)_{\alpha\beta}\,. \tag{A.2}$$

There are two identities, quartic in θ, used to show that the θ^4 terms cancel.

$$\begin{aligned}
\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^i\theta &= \frac{1}{32}\theta\gamma^a\gamma^b\theta\,\theta\gamma^i\gamma^j\gamma^a\gamma^b\gamma^j\gamma^i\theta + \frac{1}{96}\theta\gamma^a\gamma^b\gamma^c\theta\,\theta\gamma^i\gamma^j\gamma^a\gamma^b\gamma^c\gamma^j\gamma^i\theta\\
&= \frac{25}{32}\theta\gamma^a\gamma^b\theta\,\theta\gamma^a\gamma^b\theta + \frac{9}{96}\theta\gamma^a\gamma^b\gamma^c\theta\,\theta\gamma^a\gamma^b\gamma^c\theta\\
&= \frac{25}{32}\theta\gamma^a\gamma^b\theta\,\theta\gamma^a\gamma^b\theta - \frac{9}{32}\theta\gamma^a\gamma^b\theta\,\theta\gamma^a\gamma^b\theta\\
&= -\frac{1}{2}\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^i\theta\\
&= 0\,,
\end{aligned} \tag{A.3}$$

where we have used (A.2) twice. Similarly,

$$
\begin{aligned}
\theta\gamma^i\not{r}\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^i\theta &= \theta\not{r}\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^i\not{r}\theta \\
&= \frac{1}{2}\theta\gamma^a\gamma^b\theta\,\theta\not{r}\gamma^a\gamma^b\not{r}\theta \\
&= 2\theta\not{r}\gamma^i\theta\,\theta\gamma^i\not{r}\theta\,.
\end{aligned}
\tag{A.4}
$$

Calculating the θ^8 term requires a number of identities. First, some more that are quartic in θ.

$$
\begin{aligned}
\theta\gamma^i\not{r}\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta &= \frac{5}{32}\theta\gamma^{ab}\theta\,\theta\gamma^i\not{r}\gamma^{ab}\not{r}\gamma^k\theta - \frac{3}{96}\theta\gamma^{abc}\theta\,\theta\gamma^i\not{r}\gamma^{abc}\not{r}\gamma^k\theta \\
&= \frac{1}{4}\theta\gamma^{ab}\theta\,\theta\gamma^i\not{r}\gamma^{ab}\not{r}\gamma^k\theta - 3\theta\gamma^i\not{r}\theta\,\theta\not{r}\gamma^k\theta \\
&= r^2\theta\gamma^{ia}\theta\,\theta\gamma^{ak}\theta - \theta\not{r}\gamma^a\theta\,\theta\gamma^i\not{r}\gamma^a\gamma^k\theta - 3\theta\gamma^i\not{r}\theta\,\theta\not{r}\gamma^k\theta \\
&= r^2\theta\gamma^{ia}\theta\,\theta\gamma^{ak}\theta - 5\theta\gamma^i\not{r}\theta\,\theta\not{r}\gamma^k\theta + \theta\not{r}\gamma^a\theta\,\theta\gamma^a\not{r}\theta\,\delta^{ik} \\
&\quad - r^i\,\theta\not{r}\gamma^a\theta\,\theta\gamma^a\gamma^k\theta - \theta\gamma^i\gamma^a\theta\,\theta\gamma^a\not{r}\theta\,r^k\,.
\end{aligned}
\tag{A.5}
$$

Multiplying (A.5) by δ^{ik} gives (A.4). Finally,

$$
\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta = \theta\gamma^{ab}\theta\,\theta\gamma^i\gamma^{ab}\not{r}\gamma^k\theta = \theta\gamma^i\not{r}\gamma^j\theta\,\theta\gamma^j\gamma^k\theta - \theta\not{r}\gamma^a\theta\,\theta\gamma^i\gamma^a\gamma^k\theta\,. \tag{A.6}
$$

Now some that are octic in θ:

$$
\begin{aligned}
\theta\not{r}\gamma^i\theta\,\theta\gamma^i\not{r}\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta\,\theta\gamma^k\not{r}\theta &= \theta\not{r}\gamma^i\theta\left(r^2\theta\gamma^{ia}\theta\,\theta\gamma^{ak}\theta - 5\theta\gamma^i\not{r}\theta\,\theta\not{r}\gamma^k\theta + \theta\not{r}\gamma^a\theta\,\theta\gamma^a\not{r}\theta\,\delta^{ik}\right. \\
&\quad \left. - r^i\,\theta\not{r}\gamma^a\theta\,\theta\gamma^a\gamma^k\theta - \theta\gamma^i\gamma^a\theta\,\theta\gamma^a\not{r}\theta\,r^k\right)\theta\gamma^k\not{r}\theta \\
&= r^2\theta\not{r}\gamma^i\theta\,\theta\gamma^{ia}\theta\,\theta\gamma^{ak}\theta\theta\gamma^k\not{r}\theta - 4(\theta\not{r}\gamma^i\theta\,\theta\gamma^i\not{r}\theta)^2\,.
\end{aligned}
\tag{A.7}
$$

Similarly,

$$
\begin{aligned}
\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta\,\theta\gamma^k\not{r}\gamma^l\theta\,\theta\gamma^l\gamma^i\theta &= r^2\theta\gamma^i\gamma^j\theta\,\theta\gamma^{ja}\theta\,\theta\gamma^{al}\theta\theta\gamma^l\gamma^i\theta \\
&\quad - 7\theta\not{r}\gamma^i\theta\,\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{r}\theta,
\end{aligned}
\tag{A.8}
$$

$$
\begin{aligned}
\theta\gamma^i\not{r}\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta\,\theta\gamma^k\not{r}\gamma^l\theta\,\theta\gamma^l\not{r}\gamma^i\theta &= r^4\theta\gamma^i\gamma^j\theta\,\theta\gamma^{ja}\theta\,\theta\gamma^{al}\theta\theta\gamma^l\gamma^i\theta \\
&\quad - 12r^2\theta\not{r}\gamma^i\theta\,\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{r}\theta \\
&\quad + 22(\theta\not{r}\gamma^i\theta\,\theta\gamma^i\not{r}\theta)^2\,.
\end{aligned}
\tag{A.9}
$$

The identity

$$
\begin{aligned}
\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta\,\theta\gamma^k\gamma^a\gamma^i\theta\,\theta\not{r}\gamma^a\theta &= -\theta\gamma^k\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^i\theta\,\theta\gamma^i\gamma^a\gamma^k\theta\,\theta\not{r}\gamma^a\theta \\
&\quad - \theta\not{r}\gamma^b\theta\,\theta\gamma^i\gamma^b\gamma^k\theta\,\theta\gamma^k\gamma^a\gamma^i\theta\,\theta\not{r}\gamma^a\theta \\
&= -\frac{1}{2}\theta\not{r}\gamma^b\theta\,\theta\gamma^i\gamma^b\gamma^k\theta\,\theta\gamma^k\gamma^a\gamma^i\theta\,\theta\not{r}\gamma^a\theta \\
&= \theta\not{r}\gamma^b\theta\,\theta\gamma^b\gamma^k\theta\,\theta\gamma^k\gamma^a\theta\,\theta\gamma^a\not{r}\theta
\end{aligned}
\tag{A.10}
$$

is needed for

$$
\begin{aligned}
\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\not{r}\gamma^k\theta\,\theta\gamma^k\gamma^l\theta\,\theta\gamma^l\not{r}\gamma^i\theta &= r^2\theta\gamma^i\gamma^j\theta\,\theta\gamma^{ja}\theta\,\theta\gamma^{al}\theta\theta\gamma^l\gamma^i\theta \\
&\quad - 8\theta\not{r}\gamma^i\theta\,\theta\gamma^i\gamma^j\theta\,\theta\gamma^j\gamma^k\theta\,\theta\gamma^k\not{r}\theta\,.
\end{aligned}
\tag{A.11}
$$

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